

Social Welfare Functions and CBA

- CBA – Compare costs and benefits across individuals:
 - Producers
 - Consumers
 - Taxpayers
 - Third parties (external effects)

Social Welfare Functions and CBA

- Traditional CBA measures:
 - NPV, CBR, IRR, etc.
 - Add up monetary values of benefits and costs to all affected parties
 - All benefits and costs have equal weight
 - What are the implications of this assumption?
 - Makes strong assumption about the social benefits of monetary benefits/costs to different individuals in society

U_b

II

$\cdot U^1$
 $U^1 \mathbf{P} U^0?$

I

All points in Zone I
preferred to U^0

U^0

III

All points in
Zone III
inferior to U_0

IV

$\cdot U^2$
 $U^2 \mathbf{P} U^0?$

U_a

Bentham - Utilitarian

- $W = U_1 + U_2 + U_3 + \dots$
- All individuals have equal weight
- $dW = \sum_i (\delta U_i / \delta Y_i) * dY_i$
– $\delta W / \delta U_i = 1 \quad \forall i$
- In standard CBA, assume
- $(\delta U_i / \delta Y_i) = 1 \quad \forall i$
- This assumption not necessary, but then need estimates of $\delta U_i / \delta Y_i$ for all i

Kaldor - Hicks

- Kaldor – winners from a project could in principle compensate the losers from a project
- Hicks – Losers from a project cannot bribe the winners not to undertake the project
- Assumes $\delta U_i / \delta Y_i = \delta U_j / \delta Y_j$
- Or, MU(Income) is equal for all individuals
- And $\delta W / \delta U_i = \delta W / \delta U_j$

Bergson-Samuelson Social Welfare Function

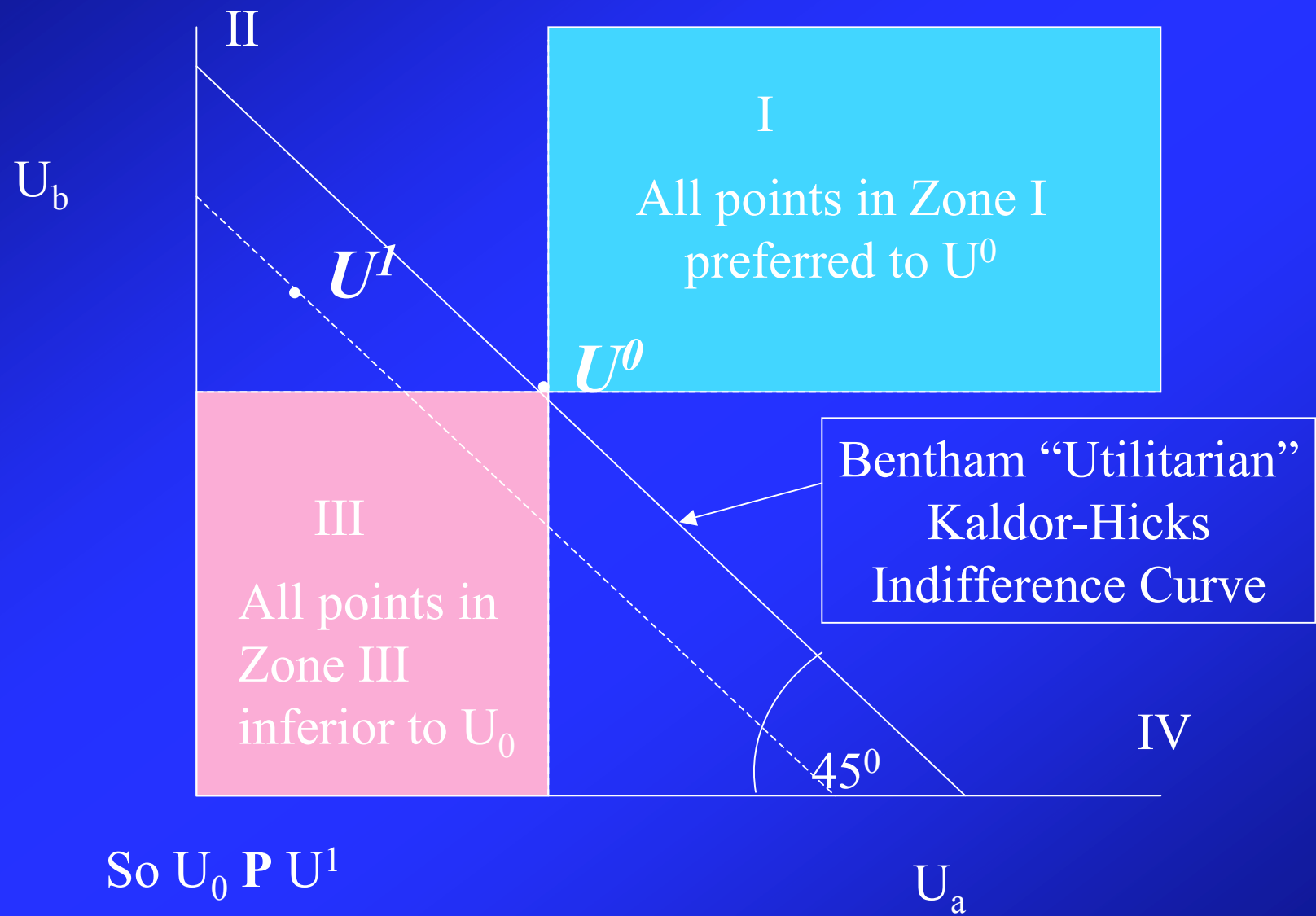
- $W = F(U_1, U_2, U_3, \dots)$
- Diminishing MRS
- $dW = \sum_i (\delta W / \delta U_i) (\delta U_i / \delta Y_i) dY_i$
- So need estimates of:
 - Marginal utility of income for all i
 - Marginal contribution to social welfare of utility for all i

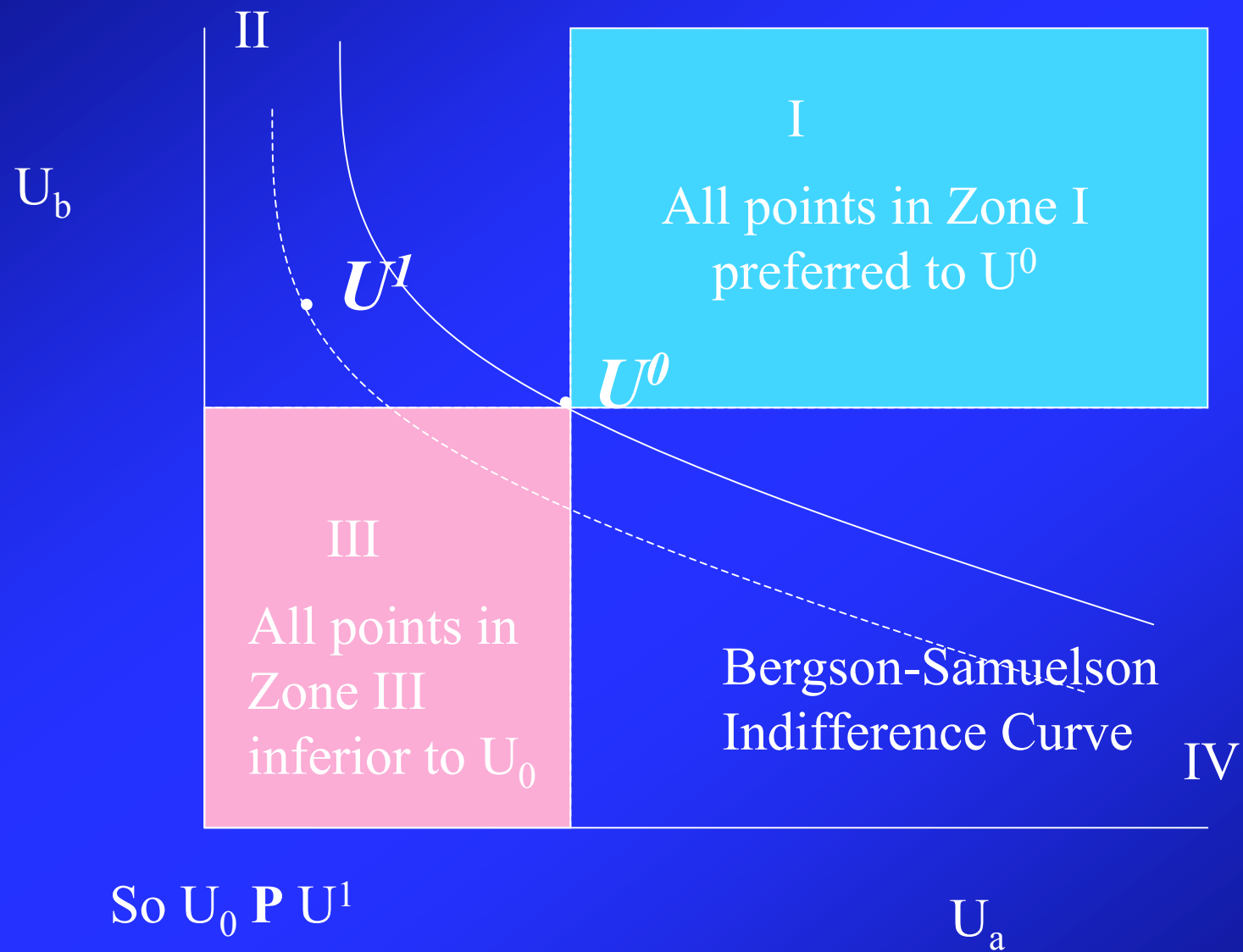
Rawls Social Welfare Function

- $W = \text{Min}_k(U_k)$
- $dW = dU_{\min}$
- Social welfare depends on utility of worst-off individual
- Moral basis – “veil of ignorance”
- Choose outcomes for all individuals in society, but the chooser does not know which individual in society he will be
- Assumes complete risk aversion

Social Welfare Functions

- Compare forms of these different Social Welfare Function forms:
 - Benthan “Utilitarian” & Kaldor/Hicks
 - Bergson – Samuelson
 - Rawls
- Compare forms of indifference curves





U_b

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$\cdot U^1$

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Rawls Indifference
Curve

V

So $U_0 \mathbf{P} U^1$

U_a

Social Welfare Functions

- Arrow Impossibility Theorem:
- Without a cardinal measure of utility (a unit of measure of utility across individuals), impossible to identify a “well-behaved” social welfare function

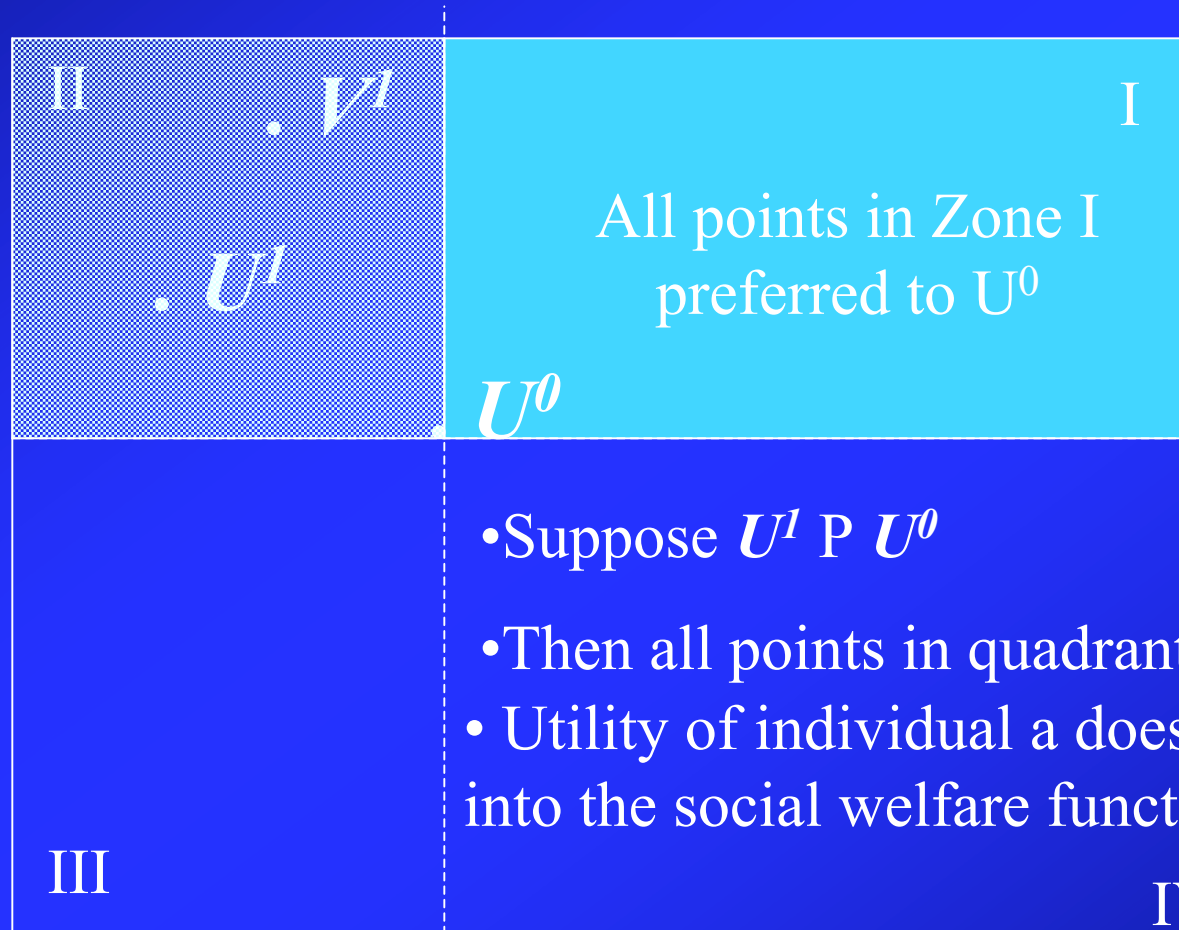
Arrow Impossibility Theorem

- Problems of aggregating welfare across individuals if utility functions can be defined only to an increasing monotonic transformation
- All monotonic transformations of a given utility functions should provide same information:
- If $u(x) > u(y)$ and $v(x) > v(y) \forall x, y$
- Then u, v are equivalent utility functions.

Arrow Impossibility Theorem

- $U_a^1 < U_a^0; U_b^1 > U_b^0$
- Any monotonic transformation of U_a, U_b will maintain same ranking, so is equivalent utility mapping
- Consider $V = \Psi(U_a, U_b)$
- Any Ψ which preserves $V_a^1 < V_a^0; V_b^1 > V_b^0$ is an equivalent mapping to U .
- So any point in quadrant II must have same preference mapping as U^1 relative to U^0

U_b



U_a

Arrow Impossibility Theorem

- Problems of identifying social preferences through voting schemes

Arrow Impossibility Theorem

	A	B	C
Smith	3	2	1
Jones	1	3	2
Arrow	2	1	3

3=most preferred, 1 = least preferred

Smith and Arrow Prefer A to B

Smith and Jones prefer B to C

Jones and Arrow prefer C to A

Arrow Impossibility Theorem

- Majority voting can lead to intransitive preferences:
- Suppose vote only on two options:
 - $A \mathbf{P} B$ (Smith and Arrow)
 - $B \mathbf{P} C$ (Smith and Jones)
 - $C \mathbf{P} A$! (Jones and Arrow)
- Also, voting cannot measure the *intensity* of individuals' preferences

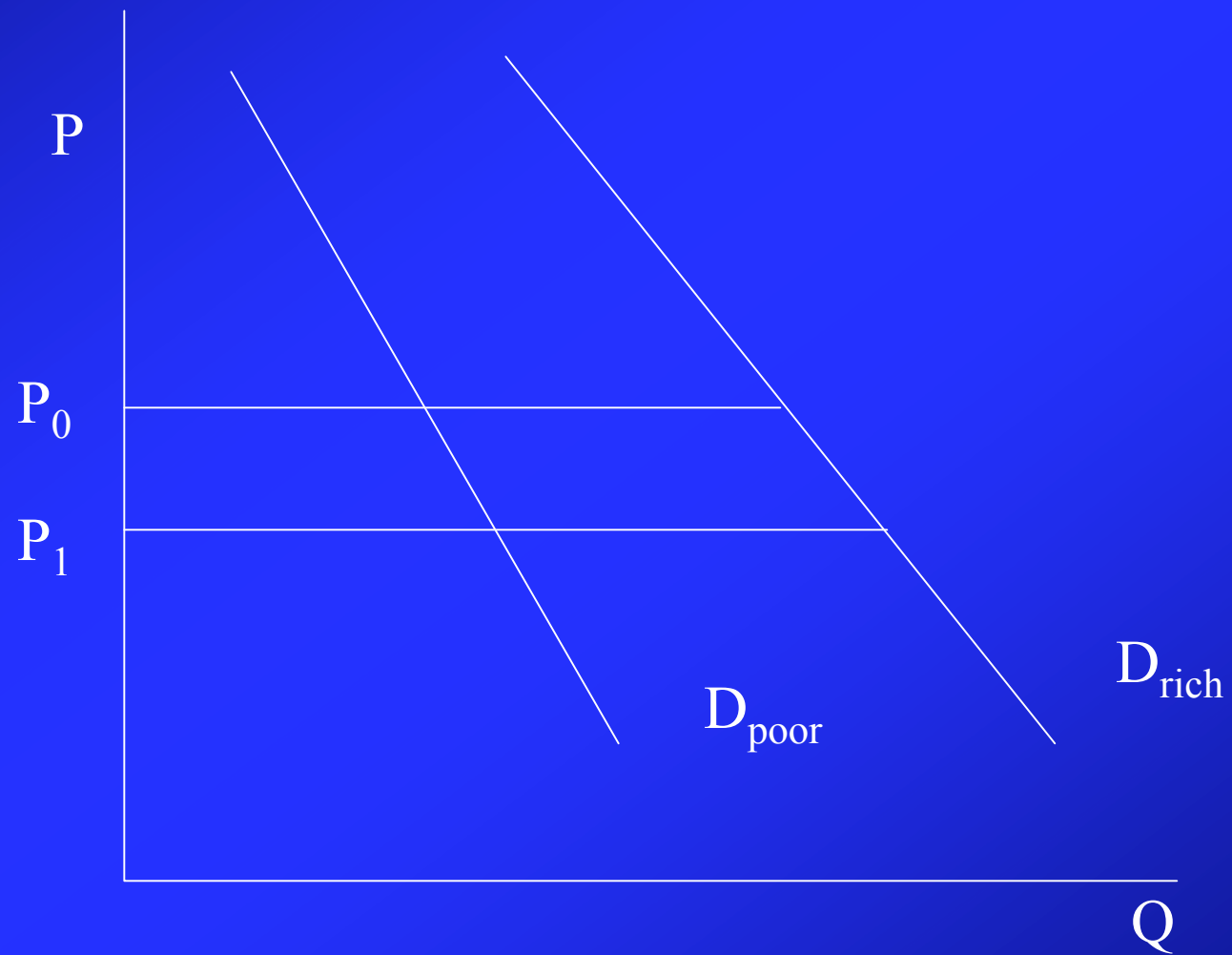
Arrow Impossibility Theorem

- Note Impossibility Problems not relevant for Rawls Social Welfare function
- Does not make inter-personal comparison
- Depends only on welfare of least well-off person
- But cannot answer many real-world problems which involve tradeoffs
- Or else, implies extreme preference for status quo

Boardman *et al.*

- Arguments for treating Low- and High-Income groups differently in CBA
 1. Diminishing MU of Income
 2. Social preference for more equal income distribution
 3. Impacts measured as changes in changes in CS or PS, rich consumers (or large firms) have more weight in the calculation

Change in CS, Rich and Poor Consumers



Reasons for weighting different income levels

Note that the arguments of:

1. Lower MU(income) of rich individuals, and
 2. Higher measured impacts of price changes
- tend to offset each other.

Social Welfare Functions

- Theoretical dilemma:
 - Cannot measure utility, so direct interpersonal comparisons are not possible
 - Without direct interpersonal comparisons, impossible to define social welfare function
- Normal procedure in CBA, assume:
 - $\delta W / \delta U_i = \delta U_i / \delta Y_i = 1$